◼ The Ackermann Award 2011

Report of the Jury

The seventh Ackermann Award will be presented at this CSL'11, held in Bergen, Norway. This is the fifth year the EACSL Ackermann Award is generously sponsored. Our sponsor for the period of 2011-2013 is the Kurt Gödel Society (KGS). Besides providing financial support for the Ackermann Award, the KGS also committed itself to inviting the receiver of the Award for a special lecture to be given in Vienna.

Eligible for the 2011 Ackermann Award were PhD dissertations in topics specified by the EACSL and LICS conferences, which were formally accepted as Ph.D. theses at a university or an equivalent institution between 1.1. 2009 and 31.12. 2010. The Jury received 10 nominations for the Ackermann Award 2011. The candidates came from 9 different countries in Europe, North America, South America and Australia, and received their degrees in 6 different countries in Europe and North America.

The topics covered the full range of Logic and Computer Science as represented by the LICS and CSL Conferences. All the submissions were of very high standard and contained outstanding results in their particular domain. The Jury wishes to congratulate all the nominated candidates for their outstanding work. The Jury encourages them to continue their scientific careers, and hopes to see more of their work in the future. The Jury decided unanimously to give the **Ackermann Award 2011** to

Benjamin Rossman.

Citation

Benjamin Rossman receives the Ackermann Award 2011 of the European Association of Computer Science Logic (EACSL) for his thesis

Average Case Complexity of Detecting Cliques.

The thesis represents a breakthrough in our understanding of circuit complexity. It settles a long-standing open question on the expressive power of first-order logic on ordered graphs and does so by developing innovative methods of proving lower bounds on the complexity of circuits. These methods advance the state of the art and represent the most significant breakthrough in circuit complexity in many years.

Background

While the main results in the thesis are in the area of circuit complexity, the motivation for the work comes from questions of logic and, ultimately, the results obtained have a strong connection with these motivating questions. Indeed, they also provide one of the most significant breakthroughs in the field of finite model theory in many years.

The motivating problem in logic is the following. Can we express more with first-order logic using $k + 1$ variables than we can with k on ordered finite graphs? This question is deceptively simple to state, but turns out to be very difficult to answer. If we drop either of the restrictions to finite or to ordered graphs, it is easy to show that an infinite hierarchy of expressive power is obtained by increasing the number of variables. On the other hand, if, instead of graphs, we consider linear orders with unary relations only, it is known that every

first-order sentence is equivalent to one with just 3 variables. The question for finite linear orders with one binary relation, i.e. finite ordered graphs, however, turns out to be tied to difficult complexity theoretic considerations.

The connections with complexity theory emerge from the work of Neil Immerman on descriptive complexity in the 1980s. Following Fagin's proof that the class of problems definable in existential second-order logic is exactly the complexity class NP, Immerman established the connection between a number of different complexity classes and corresponding definability classes. Most of these correspondences require us to assume that structures are ordered or even that they have rich arithmetic predicates available as these are necessary in order to give defining formulas the power to simulate computations. Indeed, the weaker the complexity class, the richer the fragment of arithmetic required. Thus, while NP is captured by existential second-order logic without any requirement for order, the characterization of P as definability in least fixed-point logic only works on ordered structures and AC_0 is captured by first-order logic with order *and* arithmetic predicates.

While the work on descriptive complexity had raised hopes that model-theoretic methods could be deployed to prove complexity lower bounds, the best known such methods really only provided inexpressibility results in the absence of order. Thus, the challenge before the field of finite model theory was to develop methods that could be used to establish lower bounds on *ordered* structures. An iconic problem, representing this challenge, was the question of showing that increasing the number of variables leads to an increase in expressive power on ordered finite graphs.

There is a first-order sentence with k variables that expresses that a graph contains a clique on k vertices. This sentence does not require an order. But are k variables really necessary? Or, at least, can one show that no fixed number of variables suffice to express the clique problem for all k? The question was posed in essentially this form by Immerman in 1982. It was beyond the boundary of the model-theoretic methods available yet appeared simpler than a full-fledged complexity lower bound. Indeed, it is closely connected to a question of circuit complexity. A first-order sentence ϕ with k variables can be translated to a family of circuits C_n of bounded depth (bounded by the quantifier depth of the sentence) and size n^k, such that C_n accepts encodings of those graphs of size n that satisfy ϕ. Conversely, any such family translates into a first-order sentence, but one that requires an order and arithmetic relations in addition to the graph relation in its vocabulary. Thus, Immerman's question could be answered if one could show a suitable lower bound on the size of circuits required to solve the clique problem. That is, that there is no fixed k such that circuits of bounded depth and size n^k suffice to decide the l-clique problem for all l. This was widely believed, but considered beyond the methods of circuit complexity at the time.

This lower bound is what Rossman establishes. He proves that there is no family of constant-depth circuits of size $O(n^{k/4})$ that can decide the k-clique problem. It follows that there is no sentence of first-order logic using fewer than $k/4$ variables, even with order and *arbitrary* arithmetic predicates that can express the existence of a k-clique. He then shows (using a previously unpublished construction due to Immerman) that it follows that for each k there is a sentence with $k + 1$ variables that is not equivalent to one with k.

In the 1980s, the clique problem was well-studied in the context of circuit lower bounds. Methods based on Håstad's switching lemma were used to establish a trade-off between the size and depth of circuits required. In particular, Beame showed in 1990 that any family of circuits of depth d that decided k-clique would require size $n^{\Omega(k/d^2)}$. However, it was widely held that the methods could not be extended to obtain a lower bound that was independent of depth and this is where Rossman has made a breakthrough.

Rossman's Contribution

The innovation in Rossman's method is to combine methods based on the switching lemma with considering sparse random graphs. Let $G(n, p)$ denote the probability distribution on graphs on the set of vertices $\{1, \ldots, n\}$ obtained by letting each pair (i, j) with $i < j$ have an edge with probability p. Then $p(n) = n^{-2/(k-1)}$ is the threshold for the existence of k-cliques. That is, with p much below this, the probability that a graph in $G(n, p)$ contains a k-clique goes to 0 while with p much higher than the threshold, it goes to 1. What Rossman proves is that for any family of constant depth circuits of size $O(n^{k/4})$, with high probability the same answer is obtained on a random graph in $G(n, p)$ as on one in which k-clique has been planted.

This result not only establishes a lower-bound on the worst-case complexity of the clique problem, it shows that the *average-case* complexity is worse than had been shown before. To be precise, it shows that any algorithm which can be represented as a bounded-depth family of circuits (i.e. it is sufficiently parallelizable) must take time $\Omega(n^{k/4})$ on average to decide the presence of a k-clique.

In another set of results, Rossman considers *monotone* circuits (i.e. circuits that do not use `not` gates). This is another class of circuits where lower bounds have previously been obtained. Razborov showed in 1985 that the k-clique problem cannot be decided by a family of monotone circuits of size $O((n/\log^2 n)^k)$ and the bound was subsequently further improved by Alon and Boppana. What Rossman establishes is new lower bounds on the average case complexity of monotone circuits for the clique problem. This is again done by considering sparse random graphs at the threshold for the existence of k-clique.

In some sense the results of this thesis close a line of research within finite model theory. The open problem which had inspired much interesting work has been settled. Moreover, the breakthrough has not come from extending methods from logic as had been hoped at some point but rather, it is a breakthrough in complexity theory that has settled the long-standing problem in logic. Yet, this breakthrough does provide new methods and which can and will be applied to other problems.

Among the methods that should be highlighted are a new notion of sensitivity which provides a powerful analytical tool for studying bounded-depth circuits that work on graph properties. It is this that enables Rossman to extend methods based on the switching lemma far beyond their previous use. There are also new combinatorial tools among which one should mention the quasi-sunflower lemma which provide an interesting extension of the Erdös-Renyi sunflower lemma in the "average case".

Finally, the thesis is to be commended for its presentational style, which makes difficult mathematical material so accessible.

Biographic Sketch

Benjamin Rossman received his B.A. and M.A. degrees in Mathematics from the University of Pennsylvania in 2001 and 2002 respectively. He completed his PhD in 2010 at the Massachusetts Institute of Technology under the supervision of Madhu Sudan. Since September 2010 he is at the Tokyo Institute of Technology supported by an NSF Mathematical Sciences Postdoctoral Research Fellowship. He has twice (in 2003 and 2005) received the Kleene award for best student paper at the IEEE Symposium on Logic in Computer Science. His thesis received the George M. Sprowls award at MIT.

The Jury

The Jury for the **Ackermann Award 2011** consisted of eight members, three of them ex officio, namely the president and the vice-president of EACSL, and one member of the LICS organizing committee.

The members of the Jury were

- A. Atserias (Barcelona, Spain),
- T. Coquand (Gothenburg, Sweden),
- P.-L. Curien (Paris, France),
- A. Dawar (Cambridge, U.K., Vice-president of EACSL),
- J.-P. Jouannaud (Paris, France and Beijing, China),
- D. Niwinski (Warsaw, Poland, President of EACSL),
- L. Ong (Oxford, U.K., LICS representative), and
- W. Thomas (Aachen, Germany).

They were helped by the non-voting coordinator of the Jury, J.A. Makowsky (Haifa, Israel, Secretary of the Jury and Member of the EACSL Board).

June 2011   Anuj Dawar, Johann A. Makowsky, and Damian Niwinski

The Ackermann Award

The EACSL Board decided in November 2004 to launch the EACSL Outstanding Dissertation Award for Logic in Computer Science, the **Ackermann Award**, The award is named after the eminent logician Wilhelm Ackermann (1896-1962), mostly known for the Ackermann function, a landmark contribution in early complexity theory and the study of the rate of growth of recursive functions, and for his coauthorship with D. Hilbert of the classic *Grundzüge der Theoretischen Logik*, first published in 1928. Translated early into several languages, this monograph was the most influential book in the formative years of mathematical logic. In fact, Gödel's completeness theorem proves the completeness of the system presented and proved sound by Hilbert and Ackermann. As one of the pioneers of logic, W. Ackermann left his mark in shaping logic and the theory of computation. Details concerning the Ackermann Award and a biographic sketch of W. Ackermann were published in the CSL'05 proceedings and can also be found at `http://www.eacsl.org/award.html`.

The Ackermann Award is presented to the recipients at the annual conference of the EACSL. The Jury is entitled to give more than one award per year. The award consists of a diploma, an invitation to present the thesis at the CSL conference, the publication of the abstract of the thesis and the citation in the CSL proceedings, and travel support to attend the conference.

Previous winners of the Ackermann Award

2005, Oxford:
Mikołaj Bojańczyk from Poland,
Konstantin Korovin from Russia, and
Nathan Segerlind from the USA.

2006, Szeged:
Balder ten Cate from The Netherlands, and
Stefan Milius from Germany.

2007, Lausanne:
Dietmar Berwanger from Germany and Romania,
Stéphane Lengrand from France, and
Ting Zhang from the People's Republic of China.

2008, Bertinoro:
Krishnendu Chatterjee from India.

2009, Coimbra:
Jakob Nordström from Sweden.

2010, Brno:
No award was given.

Detailed reports of the Jury, and the work of the recipients, appeared in the CSL'05, CSL'06, CSL'07, CSL'08, CSL'09 and CSL'10 proceedings, and are also available via the EACSL homepage `http://www.eacsl.org/award.html`.